

Galaxy Cluster Virial Masses and Ω

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ABSTRACT

The mean density of the universe is equal to the mass of a large galaxy cluster divided by the equivalent co-moving volume in the field from which that mass originated. To re-examine the rich cluster Ω value the CNOC Cluster Survey has observed 16 high X-ray luminosity clusters in the redshift range 0.17 to 0.55, obtaining approximately 2600 velocities in their fields. The systemic redshift, the RMS line-of-sight velocity dispersion, σ_1 , and the mean harmonic radius, r_v , are derived for each cluster using algorithms which correct for interlopers in redshift space and measure the angular extent of the sampling. The virial mass, and its internal error, are derived from these data. The cluster luminosity, corrected to $z = 0$, is estimated from the r band luminosities of the cluster galaxies. Directly adding all the light to $M_r(0) = -18.5$, about $0.2L_*$, and uniformly correcting for the light below the limit, the average mass-to-light ratio of the clusters is $283 \pm 27h \text{ M}_\odot / \text{L}_\odot$ and the average mass per galaxy is $3.5 \pm 0.4 \times 10^{12} h^{-1} \text{ M}_\odot$. The clusters are consistent with having a universal M_v/L value (within the errors of about 20%) independent of their velocity dispersion, mean color of their galaxies, blue galaxy content, redshift, or mean interior density. Using field galaxies within the same data set, with the same corrections, we find that the closure mass-to-light, ρ_c/j , is $1160 \pm 130h \text{ M}_\odot / \text{L}_\odot$ and the closure mass per galaxy, $\rho_c/\phi(> 0.2L_*)$, is $13.2 \pm 1.9 \times 10^{12} h^{-1} \text{ M}_\odot$. Under the assumptions that the galaxies are distributed like the mass and that the galaxy luminosities and numbers are statistically conserved, which these data indirectly support, $\Omega_0 = 0.20 \pm 0.04 \pm 0.09$ where the errors are, respectively, the 1σ internal and an estimate of the 1σ systematic error resulting from the luminosity normalization.

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1. Introduction

Clusters of galaxies are the largest collapsed objects in the universe and play a particularly important role in the problem of estimating the mean density of matter that participates in gravitational clustering, ρ_0 . The standard procedure is as follows. The mass of a cluster, within some radius, is measured with any of a number of techniques, such as galaxy kinematics, X-ray profiles, or gravitational lensing. To relate cluster masses to $\rho_0 = M/V$ requires an estimate of the co-moving volume, V , from which the clusters collapsed, normally done via the luminosity of the cluster galaxies, L , in ratio to the field luminosity density, j , so that $V = L/j$. The value of $\Omega_0 \equiv \rho_0/\rho_c$ can therefore be rewritten as the cluster mass-to-light ratio, M/L , divided by the closure mass-to-light ratio, $(M/L)_c \equiv \rho_c/j$. The quantity Ω_0 is independent of H_0 , and determines the future of the expansion of the universe, in the absence of a cosmological constant or a “hot” component of the mass field. The classical cluster mass estimator is the virial mass, M_v , a straightforward global estimator suitable for relatively sparse data (but see Bahcall & Tremaine 1981). This subject has a long history, with relatively stable results. That is, the virial mass-to-light ratios of clusters, M_v/L , are generally in the range $200 - 400h M_\odot/L_\odot$ (*e.g.* Zwicky 1933, Smith 1936, Zwicky 1937, Schwarzschild 1954, Gunn 1978, Ramella, Geller & Huchra 1989, David, Jones, & Forman 1995, Bahcall, Lubin & Dorman 1995) which, in ratio to $(M/L)_c \simeq 1500h$ (Efsthathiou, Ellis & Peterson 1988, Loveday *et al.* 1993) indicates $\Omega_0 \simeq 0.2$. The low value of Ω_0 from cluster M_v/L measurements is not generally accepted as a definitive measurement of the field value of Ω_0 because the random errors are generally fairly large, and there are uncontrolled systematic errors in both the mass and the light measurements which potentially allow the cluster M_v/L ratios to be consistent with $\Omega = 1$.

There are two latent problems in the M_v/L estimate of Ω . The reliability of the virial mass statistic as indicating the total gravitational mass of a cluster is critically dependent on whether the galaxy distribution traces the total mass distribution of the cluster. The large color differences between cluster and field galaxies means that their recent star formation histories are different, which may simply be a recent decline associated with infall into the cluster, or, it may indicate drastically different star formation efficiencies at early times (Dressler 1980, Dressler & Gunn 1983, Butcher & Oemler 1984, Yee *et al.* 1995) with the consequence that the luminosity per unit mass could be quite different between cluster and field. The possibility that the total mass distributions of galaxy clusters are more extended than their constituent galaxies has been recognized for many years (Limber 1959, West & Richstone 1988, Carlberg 1994). The virial mass is a usefully accurate measurement of the mass within the orbits of the galaxies, and completely independent of any anisotropy of the velocity ellipsoid, but if the cluster light is more concentrated than the cluster mass, the virial mass will be an underestimate of the total mass. This assumption of whether the mass and galaxies are similarly distributed is straightforward to test given sufficient data to derive radially resolved luminosity and velocity dispersion profiles.

Dynamical mass measurements have the benefit that their strengths and weaknesses are already relatively well understood and are easily studied further with n-body simulations.

Although not of direct interest here, dynamical mass estimation can be extended to well beyond the virialized region, by measuring peculiar velocities in the infall region, although then the results depend on the biasing of galaxies with respect to the mass field. The overall goal of this project is to measure the total mass and luminosity contained within the central virialized region of the cluster to establish a value of Ω , and measure any biases among cluster galaxies, cluster mass and field galaxies. Beyond the virialized region there is an infalling mixture of galaxies, gas, and dark matter which is very likely statistically identical to the field. Other than the few very old central galaxies and a central cD the bulk of the cluster galaxy population is likely to be composed of infallen field galaxies. Because our sample of galaxies extends from the cluster core to the distant field we can check whether galaxy population modifications only occur in the virialized region of the cluster. Therefore we will be able to empirically extend our measurement of Ω from the virialized mass distribution to the Ω of the field. The scope of this project lead to a broad collaboration under the title of Canadian Network for Observational Cosmology (CNOC).

The twin complications for interpreting velocities are that cluster galaxies and surrounding highly-correlated field galaxies are intermingled in redshift space, and, individually clusters are quite nonspherical, and internally clumpy, with the velocities at large radii being influenced by surrounding large scale structure. Hence, there are two main requirements for a successful measurement: accurate control of the background and a sufficiently large sample to average over the aspherical complications of single clusters. A feasibility study (Carlberg *et al.* 1994) showed that approximately 1000 galaxy velocities and a comparable number of field velocities distributed over 10 or so cluster fields is sufficient to indicate whether the virial mass is the total mass of the clusters. The built-in field sample allows a study of the the relative galaxy populations of cluster and field, and the closure mass-to-light and mass per galaxy.

There is no question that the cluster M_v/L technique gives, in principle, an accurate Ω value. The key issues are to constrain the technique's systematic errors and minimize its random errors. The main areas of concern are the accuracy of the virial mass and galaxy population differences between the cluster and the field. Our data allow these problems to be addressed using our sample alone, since it contains more than 1000 field galaxies, and offers the considerable benefit of homogeneous selection as discussed in Section 2. In Section 3 the cluster RMS velocity dispersions and mean harmonic radii are derived. These quantities are used to find virial masses in Section 4. The luminosity and numbers of cluster galaxies are used in Section 5 to give M_v/L and M_v/N_L ratios. The field values of these quantities are derived in Section 6, and the resulting Ω values in Section 7. A companion paper (Carlberg, Yee & Ellingson 1995) will examine the relative light and mass profiles of the clusters.

2. Sample and Observations

Clusters at moderate redshifts, $z \simeq 1/3$, have a number of advantages for mass estimation. They are sufficiently distant that they have a significant redshift interval over which the density

of foreground and background galaxies are nearly uniformly sampled in redshift. An accurate background estimate is crucial for measurements in the outskirts of the cluster profile. Clusters at higher redshifts have increasing fractions of relatively blue galaxies (Butcher & Oemler 1984) which increases the similarity between cluster galaxies and field galaxies (but they are by no means identical), and, because the mean galaxy color varies, the effect of varying galaxy types on the total luminosity can be measured.

There are a number of practical considerations which motivate our choice of clusters. At $z \simeq 1/3$, a comoving Abell diameter of $3h^{-1}$ Mpc spans an angle of about $13.3'$, which is sufficiently small that uniformity of photometry and sample selection is relatively easily assured. This size is also comparable to the field size of the Canada-France-Hawaii Telescope (CFHT) Multiple-Object-Spectrograph (MOS), approximately $10'$ square (LeFèvre *et al.* 1994). These clusters are also ideal targets for other types of mass measurement observations, such as gravitational lensing and X-ray plasma profiles.

Our cluster sample was selected from the Einstein Medium Sensitivity Survey Catalogue (Gioia *et al.* 1990, Henry *et al.* 1992, Gioia & Luppino 1994), which is a collection of serendipitous objects discovered in X-ray images taken for other purposes. For our survey we chose clusters with $z \geq 0.18$ (a consequence of the poor blue response of the CCDs then available) and in the declination range $-15 \leq \delta \leq 65$, most suitable for CFHT. To select uniformly clusters that are likely to be rich in galaxies, have a large velocity dispersion (both of which make the measurement relatively easier) and are guaranteed to have a substantial virialized component, we chose those with $L_x \geq 4 \times 10^{44}$ erg s $^{-1}$ and $f_x \geq 4 \times 10^{-13}$ erg cm $^{-2}$ s $^{-1}$. The A2390 cluster, a rich (Abell 1958), high X-ray luminosity (Ulmer *et al.* 1986) cluster was added to fill an RA gap.

Observations were made at CFHT in 24 assigned nights in 1993 January, June, October and 1994 January and March. The observational techniques and data reduction are described in Yee, Ellingson & Carlberg 1996 (hereafter YEC), and the data will be described in a series of papers (*e.g.* Yee *et al.* 1996). The fields (either EW or NS strips between $9'$ and $45'$ wide and about $8'$ high across the cluster center) were imaged with the Multi-Object Spectrograph (LeFèvre *et al.* 1994) from which Gunn g and r magnitudes are derived, and masks of spectrograph entrance slits were designed for the nonstellar images. The wavelength range of the spectra was shortened with band limiting filters chosen to match the cluster’s redshift, which typically allowed about 100 targets to be observed on a single mask. The limiting magnitude for spectroscopy for each cluster is set to optimize the number of cluster (as opposed to field) redshifts obtained. The resulting spectra are cross-correlated with a set of templates to give velocities accurate to about 100 km s $^{-1}$ in the rest frame of the cluster. The resulting “pie diagrams” for the entire sample are shown in Figures 1 and 2, where redshift increases with distance from the vertex, and the angular co-ordinate is the physical separation in the RA or Dec direction, with the zero co-ordinate being at the location of the brightest cluster galaxy (which is not necessarily at the center of the observed field).

The fraction of galaxies for which we assign slits and obtain redshifts decreases for fainter galaxies. Such a selection function, which decreases with magnitude, optimizes the rate of return of cluster redshifts, but needs to be carefully measured to reconstruct statistically the properties of the complete sample. The algorithm for deciding which galaxies will be assigned spectroscopic slits has the highest priority at the expected m_* of the cluster. The resulting $n(m)$ of the subsample with redshifts is nearly constant with magnitude, whereas the total numbers rise with magnitude. The redshift sample is corrected to the photometric catalogue with a magnitude selection weight, $w_m(m_r)$. The limiting magnitude is defined here as the m_r where $w_m = 5$. The MOS has approximately a fixed number of slits per unit sky area (about 100 in 100 square arcminutes), hence there is a geometric selection weight, $w_g(m_r, x, y)$. For quantities that depend on surface area, these weights need to be supplemented with the fraction of circle sampled by galaxies at varying radii from the chosen centre. The defining relations for these weights are described in YEC. All the results given below incorporate these weights as is appropriate.

In this paper, all distance dependent quantities are calculated assuming $H_0 = 100 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $q_0 = 0.1$, $\Lambda = 0$. The choice of q_0 is for approximate consistency with the most straightforward interpretation of our results. The dependence of distance and luminosity on q_0 largely cancels for the evaluation of Ω because it is defined relative to the field in the same redshift range. Our luminosities are corrected to a redshift zero for the sake of approximate comparability with low redshift results. These corrected quantities should only be compared to others that are adjusted to our photometric system. Hence, care should be exercised in any direct comparison of our corrected $z \sim 1/3$ quantities, for instance mass-to-light ratios, to similar quantities at $z = 0$.

3. Cluster Dynamical Parameters

The redshifts and positions of galaxies are used to define a characteristic velocity and a characteristic length scale of the cluster. As a consequence of the virial theorem, the RMS velocity dispersion has the important property that in a spherical system its value is completely independent of any variation in the shape of the velocity ellipsoid. For a triaxial object the line-of-sight RMS velocity does depend on viewing angle, which is precisely why we want to average over a dozen or so clusters. The line of sight velocity dispersion of a cluster is defined as

$$\sigma_1^2 = \left(\sum_i w_i \right)^{-1} \sum_i w_i (\Delta v_i)^2, \quad (1)$$

where the $\Delta v_i = c(z_i - \bar{z})/(1 + \bar{z})$ are the peculiar velocities in the frame of the cluster and \bar{z} is the weighted mean redshift of the cluster. The weights used in Eq. 1 are magnitude dependent geometric weights. Unweighted velocities give similar results.

The virial mass estimator normally uses the projected mean harmonic pointwise separation

$$R_H^{-1} = \left(\sum_i w_i \right)^{-2} \sum_{ij} \frac{w_i w_j}{|\mathbf{r}_i - \mathbf{r}_j|}, \quad (2)$$

where the ij sum is over all pairs. Being a pairwise quantity, R_H is sensitive to close pairs and is quite noisy (Bahcall & Tremaine 1981). Furthermore there is no straightforward way to correct R_H for the approximately rectangular “window” which encloses our cluster sample. That is, simply weighting with the fraction of a circular aperture that is enclosed within our rectangle leads to a systematic underestimate of R_H .

Here we introduce an alternate estimate of R_H . A pair of galaxies at projected co-ordinates \mathbf{r}_i and \mathbf{r}_j are statistical representatives of all galaxies at those projected radii. Under the assumption of axial symmetry the angle between these two vectors is a uniform random variable. Hence, some immediate averaging is possible by imagining that one (or the other) of the particles has its mass distributed like a ring, with a radius equal to its radial location with respect to some cluster center. More formally, the expectation value for random angles of the pairwise potential $1/|\mathbf{r}_i - \mathbf{r}_j|$ is the potential between a point and a ring, which is (noting that the result is independent of the sign of the cosine function for an integral over a circle)

$$\begin{aligned} R_h^{-1} &= \left(\sum_i w_i \right)^{-2} \sum_{ij} w_i w_j \frac{1}{2\pi} \int_0^{2\pi} \frac{d\theta}{\sqrt{r_i^2 + r_j^2 + 2r_i r_j \cos \theta}} \\ &= \left(\sum_i w_i \right)^{-2} \sum_{ij} w_i w_j \frac{2}{\pi(r_i + r_j)} K(k_{ij}), \end{aligned} \quad (3)$$

where $k_{ij}^2 = 4r_i r_j / (r_i + r_j)^2$ and $K(k)$ is the complete elliptic integral of the first kind in Legendre’s notation (Press *et al.* 1992). The quantity R_h will be referred to as the ringwise projected harmonic mean radius. Unlike the original R_H this modified R_h requires an explicit choice of the cluster center and assumes that the cluster is symmetric about the center. Neither of these pose any practical difficulties. The profile analysis will make these assumptions, but it should be noted that for flattened clusters the resulting R_h will be a small overestimate if the field sampled happens to lie along the major axis of the cluster.

There are two substantial benefits to be had from the definition of R_h in Equation 3. For close pairs the divergence is logarithmic instead of $1/r$, which makes R_h less noisy than R_H . Of immediate practical interest here is that the value of R_h is readily determined for data sets where a strip (sometimes of varying width) across the center has been sampled. For instance, any of our strips can be artificially narrowed to test how much R_h varies. For a factor of two reduction in the width of the A2390 data, R_H declines from 348 (already a substantial underestimate) to 267 arcseconds, whereas R_h drops from 533 to 517 arcseconds.

The three dimensional virial radius, r_v , is $r_v = \pi R_h / 2$ (Limber & Mathews 1960). With σ_1

and r_v the virial mass of a cluster is calculated as

$$M_v = \frac{3}{G} \sigma_1^2 r_v. \quad (4)$$

3.1. Cluster Membership

The complication in applying Equations 1 and 3 to redshift data is to decide which galaxies belong to the cluster and which are likely to be interlopers in redshift space. Because galaxy groups and other clusters have such a high clustering probability near a rich cluster this background is not expected to be smooth. Our magnitude distribution of galaxies with redshifts was designed to maximize the abundance of cluster galaxies and minimize the background. That is, the number of field galaxies per magnitude, $n(m)$, rises steeply with magnitude, $n(m) \propto 0.4m$, whereas the cluster luminosity function is much shallower, $n(m) \propto 0.25m$ or less, below m_* , the characteristic brightness of a cluster galaxy. For each cluster we estimated m_* and then set our spectroscopic exposures so that we expected to obtain redshifts down to the magnitude where cluster galaxies are starting to become less numerous than field galaxies, which typically occurs 2.5 magnitudes below m_* . The higher priorities are assigned to galaxies with brightnesses near m_* . To meet the goals of this project it is far better to have the same total number of cluster redshifts in many clusters, rather than in a single one. Besides being an efficient procedure, this strategy also ensures that the cluster has the maximum possible contrast in redshift space from the field, as shown in Figures 1 and 2. A sampling which is more complete or deeper would mainly provide more field galaxies. If more velocities are desired it is far better to increase the sample of clusters (to increase the statistical averaging over cluster to cluster variations, such as substructure) rather than decreasing the random error in a few clusters. However, fewer than 50 velocities in a cluster often leads to large uncertainties because the velocity structure of the cluster is not clear, in which case one tends to overestimate the velocity dispersion.

The calculation of the RMS velocity dispersion of a cluster uses only the redshifts with no positional information. Defining the extent of rich clusters in redshift space is a relatively difficult problem in general (*e.g.* Bird & Beers 1993, Bird 1995) because cluster members moving at a large velocity with respect to the center of the cluster are impossible to distinguish from field galaxies. Furthermore, many statistical tests to assess the membership status of galaxies in the tails of the velocity distribution are based on the assumption that the underlying distribution is a unimodal Gaussian, which is generally not true for such a kinematically complex system as an accreting cluster and the surrounding large scale structure. We have reduced the problem through the magnitude distribution of our cluster sample, which provides data that leaves clusters as cleanly defined as possible in redshift space.

The procedure for defining the redshift range of the cluster is a manually iterated procedure, roughly equivalent to measuring the width of an emission line in a spectrum. First, an initial estimate of the outer limits of the redshift range of the cluster is made, using Figures 1, 2, and

3. The redshift limits adopted are shown as dotted lines in Figure 3. The initial estimate of the redshift range is used to calculate a trial velocity dispersion, σ_1 . A second algorithm is used to assess the validity of this velocity dispersion in the presence of interlopers in the outer parts of the redshift space of the cluster. The velocity dispersion validation algorithm works as follows. The weighted (as in Eq. 1) velocities within $15\sigma_1$ (or the edge of the band-limiting filter if that is smaller) of the cluster center are put into bins $0.1\sigma_1$ wide (within reason the bin width has no effect). The mean density of the background in velocity space is calculated from the mean bin density $> 5\sigma_1$ away from the cluster. This background is then subtracted from the data within $3\sigma_1$ of the cluster center and the velocity dispersion σ'_1 is calculated, along with a Bootstrap estimate of the error in σ'_1 . If σ'_1 and σ_1 are within one standard deviation, σ_1 is accepted as the cluster’s velocity dispersion; if they are not equal, the redshift limits are further adjusted.

The adopted ranges in redshift, as defined by the above procedure, and radius, as defined by the rectangular region which bounds the data, are given in Table 1. Column 2 gives the weighted mean redshift, columns 3 and 4 gives the dimensions of the bounding box in RA and Dec as converted to physical lengths, $D_A(z)\Delta\theta$, at the redshift of the cluster. Columns 5 and 6 give the lower and upper redshift boundaries. In Table 2 the second column gives the ratio of the “unlimited” σ'_1 to the value accepted, and the third column gives the 1 standard deviation confidence interval for the σ'_1/σ_1 ratio. Each cluster’s velocity dispersion is calculated using Equation 1 with the weighted data in the cluster’s redshift range given in Table 1. The results are given in Table 3. Column 2 of Table 3 is the redshift, column 3 is the number of galaxies above the magnitude limit in the cluster RA, Dec and redshift range. Column 6 is σ_1 . The sky positions of the cluster galaxies for the inner 1000 arcsec of the sample are shown in Figure 4.

The obvious redshift limits were acceptable in about half of the clusters. In the other half the limits were adjusted until a statistically acceptable result was found, or until no stable value emerged. The velocity dispersion checking algorithm indicates (Table 2) that the velocity dispersion of MS0906+11 is significantly too large. No consistent solution could be found for this cluster, which appears to be an indistinct binary in redshift space. All other clusters have velocity dispersions that are well within two standard deviations of the input values. Based on this test, we will accept the results of Table 3 as our standard values. One could apply the slight corrections of Table 2, but that is simply changing the values within their standard error. The cluster MS1358+62 appears to be an unequal binary in declination (Fig. 2), although its velocity dispersion appears to be acceptable.

The modified mean harmonic radii, R_h , are calculated for each cluster using the entire angular extent of the available data in the redshift range of the cluster as fixed for the σ_1 calculation. The mean interior overdensity inside r_v is an indicator of the radial range of the cluster that has been observed. The angular extent of the clusters is defined by the angular limits on the sky of our sample.

It is expected (Gott & Gunn 1972) and borne out by numerical simulation (*e.g.* Crone,

Evrard & Richstone 1994) that, on the average, substantially all the virialized cluster mass is contained inside the radius where $\bar{\rho} > 200\rho_c$. Outside that radius the velocities very rapidly become dominated by infall. Therefore the ratio $\bar{\rho}(r_v)/\rho_c$ is used as a test of the completeness of the radial extent of the sampling of the cluster. This ratio is calculated as the mean density inside r_v to the critical density at the observed redshift by dividing the virial mass (Eq. 4) by the volume inside r_v ,

$$\begin{aligned} \frac{\bar{\rho}(r_v)}{\rho_c(z)} &= \frac{1}{\rho_c(z)} \frac{3M_v}{4\pi r_v^3} \\ &= \frac{6}{(1+z)^2(1+\Omega_0 z)} \frac{\sigma_1^2}{H_0^2 r_v^2}. \end{aligned} \quad (5)$$

The critical density at redshift z is $\rho_c(z) = \rho_c(0)(1+z)^2(1+\Omega_0 z)$, which for $\Omega_0 = 1$, has the $(1+z)^3$ dependence of the mean density. For $\Omega_0 < 1$ Eq. 5 means that the density ratio at z is somewhat larger than for $\Omega_0 = 1$, with the difference being 25% at $z = 1/3$ for our adopted $2q_0 = \Omega_0 = 0.2$.

The *total* virialized mass of the cluster should be accurately estimated if the mean density inside r_v is $200\rho_c$ or less. Table 3 includes 6 clusters for which this criterion is met. For reference, the Coma cluster has $R_h \simeq 1.78h^{-1}$ Mpc (1.48° , Schwarzschild 1954) and, with σ_1 of 1040 km s^{-1} (Peebles 1970) has $\bar{\rho}(r_v) = 77\rho_c$.

3.2. Error Analysis

The errors in σ_1 , R_h , M_v , and M_v/L are assessed using the Jackknife technique. This is one of the simplest resampling techniques, wherein partial standard deviations, δ_i , are calculated by taking the difference between the $f(x_1, \dots, x_n)$, where f is any statistical quantity calculated from the entire data set, and the same quantity calculated dropping one element of the data set, $\delta_i = f(x_1, \dots, x_n) - f(x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n)$. The estimate of the variance is $\sqrt{n/(n-1) \sum_i \delta_i^2}$ (Efron 1981, Efron & Tibshirani 1986). For a Gaussian distributed sample the Jackknife variance converges to the normal value. A closely related technique is the Bootstrap, in which randomly drawing *with replacement* from the original data set creates many new data sets of the same size as the original which are then analyzed in precisely the same manner as the original to give a distribution of results from which confidence intervals are calculated. A significant difference is that the Bootstrap data sets have repeated values, which for pairwise statistics gives singularities. Also, the Bootstrap can be computationally quite expensive, since tests indicate (Efron & Tibshirani 1986) that 300 or more resamplings are required to give solid results. The errors given in Table 3 (in the columns labeled ϵ_r and ϵ_σ) are approximately in accord with a \sqrt{N} expectation, but have significant differences in detail. The reliability of these errors is extremely important for one of the major results of this paper. We believe the pragmatic nature of the Jackknife error estimate is well suited to the task.

3.3. Sample Commentary

A cluster-by-cluster comparison with the results of other workers is done in the data papers for each cluster. In a number of cases our velocity dispersions are notably lower than those previously found. For instance, we find for Abell 2390 that $\sigma_1 = 1100 \pm 63 \text{ km s}^{-1}$ whereas a previous study in the central region found $2112_{-197}^{+274} \text{ km s}^{-1}$ (Le Borgne *et al.* 1991). In general the differences from previous work can be attributed to three factors. First, the precision of our velocities, $\simeq 100 \text{ km s}^{-1}$, reduces catastrophic velocity errors and helps identify significant substructures. Second, our data cover a large radial range in the cluster, which makes them less subject to local substructure, and more representative of the RMS value. Third, the bluer galaxies, which often contain measurable emission lines, statistically are found to have a higher velocity dispersion than the redder absorption line galaxies, an effect which is particularly prominent near the projected center of the cluster (Abraham *et al.* 1995, Carlberg, Yee & Ellingson 1995). Since redshifts are easily measured from emission lines this can lead to an upward bias in the velocity dispersion, but which is demonstrated to be quite small in our data (Yee, Ellingson & Carlberg 1996). These effects can be deadly in the presence of substructure, for which there is evidence in many of our clusters. In general the clusters are quite regular, possibly connected to their high X-ray luminosity, and usually does not upset any of the results below. The exceptions are identified and removed, as discussed below.

The quantity $\bar{\rho}(r_v)$ in Table 3 varies from a low of $84\rho_c$ in MS1621+26 to $1135\rho_c$ in MS1455+22, discounting MS0906+11. It is clear that $\bar{\rho}(r_v)$ is high for those clusters for which only a central field was observed. In particular, we expect on the basis of Coma data and n-body simulations, that it is necessary (but not sufficient) that $\bar{\rho}(r_v)$ should be less than $200\rho_c$ if the formal M_v (Eq. 4) is interpreted as the total virialized mass. If $\bar{\rho}(r_v) > 200\rho_c$ then it indicates that r_v is an underestimate of the true value for the entire virialized system, which is a straightforward consequence of the limited angular sampling of some of our clusters. This undersampling is an important issue, but is not a significant problem for the major goals of this paper because some quantities, such as M_v/L , are insensitive to the outer limits of the sampling, and it is possible to define a characteristic radius at a fixed mean interior density, which is insensitive, within reasonable limits, to these sampling variations.

4. Virial Mass Analysis

The quantities σ_1 and r_v are given in Table 3. The resulting M_v (Eq. 4) are given in Table 4, where column 2 repeats the $\bar{\rho}(r_v)$ column of Table 3, column 3 is M_v in units of solar masses, column 4 is the total observed luminosity in units of solar luminosities (see Section 5), columns 5 and 6 are the canonical M_v/L ratio and its standard error from the Jackknife technique. It must be borne in mind that the masses given are defined by the radial extent of the sample, and must be referred to the mean density given in column 2. That is, if the cluster extends beyond the size

of our observed field, our mass is an underestimate of the total mass. Nevertheless, it remains of considerable interest to compute an M_v/L for the part of the cluster we observe. Furthermore it is relatively straightforward to bring all the masses to a common ground.

The virial mass, as calculated from a data set which misses the outer parts of the cluster, will still accurately estimate the mass contained within the orbits of the galaxies observed, with a weak dependence on the radial extent of the sampling as we show here. The virial theorem is derived by multiplying the Jeans equation of stellar “hydrostatic” equilibrium (a vector equation) with co-ordinate vectors and integrating over the volume of the system (*e.g.* Binney & Tremaine 1987). The outer limit of the volume integral is usually taken at infinity, but, when evaluated for a finite subset of the entire volume, gives an additional term to be evaluated at the outer surface. The scalar virial theorem for a spherically symmetric system bounded at radius r_b becomes

$$\int_0^{r_b} (\sigma_r^2 + 2\sigma_\theta^2) \rho dV + \int_0^{r_b} \frac{-GM(r)}{r} \rho dV = 4\pi\rho\sigma_r^2 r_b^3. \quad (6)$$

The right hand side of Eq. 6 is sometimes called the $3PV$ surface term. The effect of ignoring the surface term for a gravitating system is to overestimate the mass of the system, since the surface pressure reduces the amount of mass to keep the system in equilibrium.

A singular isothermal sphere gives an estimate of the largest reasonable error if we use Eq. 4 on a partially sampled cluster. The isothermal sphere provides an upper limit, since equilibrium stellar systems normally have a falling velocity dispersion with radius, so the isothermal sphere has an unusually large surface term. The density profile of a singular, isotropic, isothermal sphere of velocity dispersion $\sigma = \sigma_r$ is $\rho = (2\pi G)^{-1} \sigma^2 r^{-2}$. Equation 6 then becomes

$$3\sigma^2 M_b - \frac{GM_b^2}{r_b} = \sigma^2 M_b, \quad (7)$$

where M_b is the mass inside the boundary. If the surface term is neglected, then the derived virial mass will be an overestimate of the mass contained inside r_b , $3G^{-1}\sigma^2 r_b$, instead of the correct $2G^{-1}\sigma^2 r_b$. If the density profile declines more rapidly than r^{-2} , as is indeed the case for the clusters studied here, then the error introduced through the neglect of the surface term is reduced. The isothermal sphere has the pleasing property that the relative size of the surface term is completely independent of shape or size of the surface. We conclude that if only an inner part of the cluster profile is sampled, then the calculation of the virial mass will be at most a 50% overestimate.

Under the assumption that clusters are approximately $\rho(r) \propto r^{-2}$ (or any other mass profile of interest), the masses can then be cautiously extrapolated to other radii or mean interior densities. The results of extrapolating the masses to a constant mean interior density of $200\rho_c$ using $M_{200} = M_v \sqrt{\bar{\rho}(r_v)/(200\rho_c)}$ are given in the second last column of Table 4.

In “raw” M_v the most massive cluster in the sample is A2390 (discounting MS0906+11), with $M_v = 1.7 \times 10^{15} h^{-1} M_\odot$, which makes it comparable to Coma’s $2.1 \times 10^{15} h^{-1} M_\odot$. Coma and

A2390 have similar mean interior overdensities at r_v , 77 and 110, respectively, putting the mass comparison on nearly the same density basis. The masses of the other clusters range a factor of about 2.5 bigger and smaller than $M_{200} \sim 5 \times 10^{14} M_\odot$. This rather narrow range in mass is a consequence of our X-ray luminosity and flux selection. It is perhaps not too surprising that one of the most massive clusters, MS0016+16, is also the highest redshift one in our sample.

5. Cluster Galaxy Luminosities and Numbers

There are two practical approaches to relate empirically the cluster contents to a co-moving volume in the field. A third approach, a total cluster baryon inventory (White *et al.* 1993), cannot at present directly relate the cluster to the field since most field baryons are in some unobserved form and one must turn to a Big Bang Nucleosynthesis argument (Walker *et al.* 1991) to obtain a field density of the baryons. However, the large reservoir of non-stellar baryons in clusters raises concern about the assumption that the fractional conversion of gas into stars has been the same in clusters and the field. Our galaxy sample extends from cluster to the far field, allowing a number of tests for variation. The two “binary” clusters, MS0906+11 and MS1358+62 are excluded from all averages calculated below.

The total luminosity of a cluster estimates the co-moving volume from which it collapsed, $V = L/j$. This approach assumes that the total luminosity is conserved, or that any systematic change in galaxy luminosities during the gravitational assembly of the cluster can be measured and corrected. However the subject of galaxy evolution within clusters remains quite controversial, at the very least because the observational phenomena are not yet well defined.

An alternate approach to measuring the equivalent field co-moving volume of the cluster is to use the numbers of cluster galaxies above some absolute magnitude in ratio to the field, $V = \rho_c / \phi(> L)$. Each of these methods has different strengths and weaknesses, but they are substantially *independent* since cluster total luminosities are dominated by the brightest galaxies but the total numbers are dominated by the low luminosity galaxies. One possible evolutionary effect is the merging of the stellar components of two galaxies which makes no difference to the total luminosity if there is no accompanying episode of star formation. Merging does, of course, decrease the numbers of galaxies. It is well known that cluster galaxies are on the average significantly redder than field galaxies (Figure 5), which is taken to indicate that star formation has decreased in cluster galaxies and hence they are less luminous per unit stellar mass than in the field. Fading will decrease the numbers of cluster galaxies above some fixed luminosity limit relative to the field, although, if the limit is well below M_* , then the decreased numbers will be a small fraction of the change in luminosity. For instance, if all cluster galaxies down to $M_r(0) = -18.5$ were included in the total count, but there had been a one magnitude fading with respect to the field, then galaxies down to -17.5 should have been included in the total count. The expected increase in numbers (for a Schechter function with $M_* = -20.3$ and $\alpha = 1.25$) would be only 17%, as compared to a 150% correction required to the luminosity. Cluster galaxies are

not simply post-star formation field galaxies with diminished luminosities, since clusters contain a population of very luminous, but very red, Elliptical and cD galaxies in their centers. These may have been created through mergers, or they could have tapped the large baryon reservoir present in the cluster to create more stars. If galaxy luminosities are either increased or decreased, the L of a cluster responds in proportion, but the numbers of cluster galaxies above some magnitude change far less. The evolution of cluster galaxies relative to field galaxies, especially at these redshifts, is sufficiently ill defined that we do not, at this stage, want to make a highly uncertain differential correction. However, we will use mass-to-light and mass-per-galaxy to obtain two substantially independent estimates of Ω , which in turn will be used to address the notoriously difficult problem of constraining the size of systematic errors. That is, if the two estimators give discrepant values beyond the errors, then we would have detected a systematic effect (whose source would then be identified and corrected, if possible). We cannot empirically detect systematic errors smaller than our random errors which then becomes our systematic error estimate.

The observed luminosity of the cluster galaxies, summed to $M_r(0) = -18.5$, corrected as described below, is reported in Table 4. The luminosity is measured in the Gunn r band, using $M_r(\odot) = 4.83$. We also note that r band selection has the useful benefit that it is not unduly sensitive to star formation increases, although these are readily detected in the $g - r$ colors. The measured m_r are K corrected using the following method. Model K-corrections in r and $g - r$ colors as a function of redshift for non-evolving galaxies of 4 spectral types (E+S0, Sbc, Scd, and Im) are derived by convolving filter response functions with spectral energy distributions from Coleman, Wu & Weedman (1980). These values are then corrected from the AB system to the standard Gunn system (Thuan & Gunn 1976). For each galaxy with redshift, a spectral classification is estimated by comparing the observed $g - r$ color with the model colors at the same redshift. The spectral classification, obtained via interpolation, is treated as a continuous variable within the 4 spectral types. From the spectral classification, the appropriate K-correction to the r magnitude is then derived using the models. Our data strongly indicates that the K corrected cluster luminosity function is brightening with redshift (Yee, Ellingson & Carlberg 1996), hence our luminosities are given a mild evolutionary correction, $E(z) = -z$ (Yee & Green 1984), so that they are appropriately faded to the current epoch.

The corrected magnitudes and colors are shown in Figure 5. An entirely empirical redshift normalization of the colors, $(g - r)_z = (g - r)/(1.2 + 2.33 * (z - 0.3))$, is shown. This relation is based on a linear fit to the redshift dependence of the red sequence of the bright cluster galaxies. Because the median color of these clusters is always quite red, there is little range in the quantity $(g - r)_z$. The fraction of the galaxies that are blue can be estimated with a parameter similar, but quite different in detail, to the Butcher-Oemler blue fraction (Butcher & Oemler 1984). The quantity F_b here is the fractional weight of all galaxies bluer than $(g - r)_z = 0.7$. Note that this includes all galaxies used in the total luminosity and does not restrict the measurement of the blue fraction to some fiducial region. A more detailed analysis of the Butcher-Oemler effect will be presented elsewhere (a preliminary account is in Yee *et al.* 1995).

It is conventional to quote M_v/L values which are based on the “total light”, that is, a correction is made for the part of the luminosity function below the limiting magnitude. This is not strictly necessary here, since the field sample is treated in precisely the same manner, but for the sake of comparability we do this correction, for both the cluster and field luminosities. The “unobserved light” beyond the limiting magnitude our sample is estimated assuming $M_* = -20.3 + 5 \log h$ and $\alpha = -1.25$ in a Schechter luminosity function. A useful limiting magnitude is about $M_r = -18.5$, which is about $0.2L_*$. The figures and tables of this paper use this limiting luminosity, unless otherwise stated, which helps put all the M_v/L values on the same basis. There is very little difference in the estimated L ’s if we include all the galaxies to the sampling limit. The Ω estimate from the M_v/L has no dependence on the correction to the total luminosity, since the field galaxies will have precisely the same multiplicative correction included.

The luminosities, like the masses, are the total luminosities for the sample, and must be referred to its mean interior density. The luminosities can be extrapolated to a common mean density, L_{200} , in the same manner as the masses. That is, $L_{200} = L \sqrt{\bar{\rho}(r_v)/(200\rho_c)}$, which builds in a correlation between L_{200} and $\sigma_1^{1/3}$, but not with the dynamical range observed here. The cluster quantities are defined on the basis of a constant ratio with respect to the critical density at the redshift of the cluster, $\rho_c(z)$, which increases with redshift. Since $\sigma^2 \propto \rho^{1/3} M^{2/3}$, with the redshift dependence of ρ included this becomes $\sigma_1 \propto (1+z)^{1/3} (1 + \Omega_0 z)^{1/6} M^{1/3}$, Figure 6 plots the redshift adjusted luminosity against the velocity dispersion. This plot excludes MS0906+11 and MS1358+62.

5.1. Cluster Mass-to-Light Ratios

It is immediately apparent from Table 4 that the M_v/L do not show much variation outside their errors, in spite of the variations in the sampling radii. The mean M_v/L for the sample (excluding MS0906+11 and MS1358+62) is $283 \pm 27h M_\odot/L_\odot$ (in Gunn r , corrected to $z = 0$ as described above), and $245 \pm 32h M_\odot/L_\odot$ for the six clusters that have a mean interior density less than $200\rho_c$, excluding the asymmetric MS1358+62 cluster. The M_v/L values as a function of redshift are shown in Figure 7. It should be noted that luminosity evolution has been included, at the rate of $\Delta M = -z$. In the absence of this correction the luminosities at $z = 0.55$ would be 1.66 times higher, and would lead to a noticeable gradient in the diagram. The corrections to the luminosities are all multiplicative factors that are applied both to the field and the cluster data, so they have no effect on the Ω value derived.

The distribution of M_v/L differences from the 14 cluster sample mean, normalized to their measurement errors, is also displayed in Figure 7. The cluster at -3.6 standard deviations, MS1621+26, is one of the clusters with the best data and is also one of the bluest clusters, possibly indicates that its luminosity is higher than it would be if the galaxies were allowed to age enough that the colors became comparable to other clusters in this redshift range. The redshift- M_v/L plot of Figure 7 also gives the impression that all the clusters have the same M_v/L

(removing MS0906+11 and MS1358+62) within the errors. The χ^2 per degree-of-freedom is 1.8, but removing MS1621+26 as well reduces it to 1.03 which is consistent with no variation beyond the errors. That is, the M_v/L of Table 4 are consistent with a universal underlying M_v/L , after a 3 standard deviation clipping is applied. This means that the measured cluster light is a very good indicator of the mass contained within the orbits of the galaxies. It is intriguing to note that this appears to be true over a substantial range of mean interior densities which characterize the cluster samples. A similar result has been found for X-ray measurement of cluster masses (David, Jones, & Forman 1995), although their M/L profiles values decline with increasing overdensity, whereas our integrated values are nearly constant, or rise slightly, with mean interior overdensity, within the limits expected from the virial surface term of Eq. 7.

The percentage errors in the M_v/L values are in the 10-30% range, depending on the number of cluster galaxies. With more data it seems quite likely that it will become evident that there are intrinsic variations in cluster M_v/L values. In particular the cluster light could have a “second order” luminosity correction which would depend on the precise mix of galaxies in the cluster. This is already the case with MS1621+26 which has 98 members with redshifts and is somewhat “blue” compared to the trend of colors with redshift. Of course any true M_v/L variations cannot be bigger than the 20-30% error bounds found here.

5.2. Cluster Mass-to-Number Ratios

The mass per galaxy for all galaxies brighter than some chosen absolute magnitude is less sensitive to differential fading (or possible brightening) between the field and cluster than the mass per unit luminosity. Merging could alter the galaxy numbers, but, in as much as mergers lead to ellipticals (the E+A galaxies appear to be mostly disk systems, Dressler *et al.* 1994), it does not drastically alter the numbers of galaxies. Both fading and merging will cause the M_v/N_L ratio to overestimate the value of Ω . The individual mass-to-number ratios to a limiting magnitude of $M_r(0) = -18.5$ are shown in Figure 8 and given to -18.5 and -19.5 in Table 5. The χ^2 for a limit of -18.5 is very large, about 3 per degree of freedom. At a limit of -19.5 the χ^2 per degree of freedom drops to 0.6 if MS1621+ 26 is dropped.

Like M_v/L , M_v/N shows no significant dependence on other cluster parameters. Figure 8 weakly suggests an increase in scatter as a function of the mean interior overdensity, although this is likely to be primarily a statistical effect, since the highest density clusters are those that are least well sampled. There is a tendency for the high velocity dispersion clusters to have the largest M/N_L values. Although this trend is not very significant, it is opposite to a standard biasing scheme, “peak-background”, in which galaxy formation is initiated earlier and is more advanced in clusters of increasing mass.

The average values of M/N_L for all the clusters are given for a range of limiting magnitudes in Table 6. The table demonstrates that the M/N_L is not very sensitive to the limiting absolute

magnitude if it is at least 2 magnitudes less than M_* , but that the sensitivity rises quickly (exponentially is expected for a Schechter luminosity function) near M_* . The reduced χ^2 of the deviations from the mean are very large for low luminosity limits. In the case of a limit of $M_r(0) = -19.5$ the reduced χ^2 drops to 0.8. This is partially a consequence of the smaller number of cluster galaxies, typically 60% of the full sample, leading to an increase in the random errors. None the less, the reduction in χ^2 is so striking that it suggests that the most luminous cluster galaxies have far less cluster-to-cluster variation than the lower luminosity ones, which is a comment that has been made in other contexts.

6. Light and Number Densities of Field Galaxies

The closure mass-to-light ratio is defined as $(M/L)_c \equiv \rho_c/j$, where $\rho_c = 3H_0^2/(8\pi G)$, and $j = \int_0^\infty L\phi(L) dL$, where $\phi(L)$ is the field luminosity function. The integration is carried to low luminosities by assuming a Schechter form for the luminosity function for galaxies below the absolute magnitude cutoff. Similarly the closure $(M/N_L)_c \equiv \rho_c/\phi(> L)$, where N_L is the number of galaxies more luminous than L . At low redshift $(M/L)_c$ is estimated to be about $1500^{+700}_{-400} h M_\odot/L_\odot$ (Efsthathiou, Ellis & Peterson 1988, Loveday *et al.* 1993). This value is based on a blue selected sample, and integrates all the light in the luminosity function to very faint magnitudes. Our selection is based on Gunn r , and we do not sample galaxies much fainter than $M_r \simeq -18.5$. These differences would lead to substantial systematic errors in Ω if we tried to ratio moderate redshift cluster light to a low redshift field estimate of the closure density.

Our survey includes a built-in field sample which can be used to estimate accurately $(M/L)_c$ and $(M/N)_c$, on the same manner as our cluster values. Each field galaxy is K corrected and evolution corrected, and within a redshift range we correct the total luminosity for light below our magnitude limit in precisely the same as the cluster galaxies. Because all of these factors are multiplicative, they cancel in the eventual ratio of interest for Ω . The corrections are done for the sake of providing numerical values having some degree of comparability to true low redshift quantities. The luminosity density of the field can be nearly trivially calculated from our field sample, and offers the benefits that it is selected in precisely the same way, in the same filter band, in the same redshift range, and to the same limiting magnitude as the cluster data. This approach, using a self-consistent luminosity system for the cluster and field, greatly reduces concerns about systematic differences between the low redshift estimate and our medium redshift data.

Table 7 gives the results of the $(M/L)_c$ and $(M/N_L)_c$ calculation to the indicated limiting magnitudes, averaged over the redshift range $0.2 \leq z \leq 0.6$. The cluster redshift ranges of Table 1 are widened by $\Delta z = 0.01$ at both the upper and lower redshift to eliminate any concerns about an overestimate of j from a higher density of galaxies in the vicinity of these very rich clusters. In fact there is no significant change in the field luminosity density without this extra cut around the clusters. The weighted luminosities of the field galaxies, corrected to $z = 0$ precisely as the cluster data, are simply summed in each redshift bin, cutting the sum off at various absolute magnitudes.

The volumes are in *co-moving* $h^{-1} \text{Mpc}^3$ integrated over the accessible redshift range of the bin for our chosen q_0 , times the total solid angle of our survey, 2367 square arcminutes. There is no statistically significant redshift trend in either the luminosity or number density in our evolution corrected magnitude system. If evolution correction were not included the values would be about 35% smaller, and an evolutionary trend would be marginally significant in the luminosity density.

The random errors of our closure $(M/L)_c$ (column 4 of Table 7) are calculated using the Jackknife technique assuming that each cluster field should be treated as a data element. This might be a conservative estimate of the errors, but fully accounts for large scale structure variations.

The colors of the field galaxies are shown in Figure 5. The field galaxies are always substantially bluer than the cluster galaxies: the average $(g - r)_z$ is 0.55, with no significant change with redshift over the entire range of $0.1 \leq z \leq 0.6$. The cluster galaxies have average $(g - r)_z$ values ranging from 0.72 to 0.94. Although there is some overlap of the populations, there is no overlap of the averages. Hence it is not possible to empirically estimate from this sample the luminosities of the cluster galaxies if they had the same colors as the field galaxies.

7. Ω

The value of Ω is estimated under the assumption that the M_v/L of the clusters is the same as M/L in the field. Our best estimate, in the sense of smallest random errors is the M_v/L estimate to a limiting magnitude of -18.5 for which we find that $\Omega = 0.25 \pm 0.05$ at a mean redshift of 0.31. Brighter absolute magnitude limits give consistent values of Ω , but with increased errors.

The value of Ω from the M_v/N_L calculation is 0.28 ± 0.08 , for absolute magnitude limits of both $M_r(0) = -18.5$ and -19.5 . This value is consistent with the one found from the M_v/L argument. We will prefer the M_v/L value of Ω mainly because of its smaller errors, but also as the conventional approach. The fact that the M_v/L and the M_v/N_L routes to calculating Ω gives similar results is considerable grounds for confidence in the result. The two Ω values have substantially different dependencies on the evolution of cluster galaxies, and are weighted to different parts of the luminosity function. Certainly we would not be able to detect any systematic errors in cluster galaxy evolution which is smaller than the quadrature summed random errors of the two Ω results. We will use this error sum as our estimate of the likely size of any residual systematic errors in our cluster Ω values, due to galaxy evolution alone. It does not measure any systematic dynamical errors in the masses.

Tables 6 and 7 both have a common trend M/L values where fainter imposed cutoffs, which use more of the real data, find less light than the adopted luminosity function does, although the trend is not very significant.

Since $\Omega_0 = \Omega_z / (1 + z(1 - \Omega_z))$, for our average redshift of 0.31 this is $\Omega_0 = 0.20 \pm 0.04$, where the formal 1 standard deviation random error is given. The strength of this value is that

is entirely from our survey alone, with its well understood sampling statistics, requires no low redshift comparisons, and is done entirely within one photometric system which requires only small redshift corrections. The main issue which is addressed in the companion paper (Carlberg, Yee & Ellingson 1995) is whether the virial mass correctly estimates the total mass of clusters, with further examination of the differences between cluster and field light per unit mass.

8. Conclusions

The CNOC cluster sample is a uniform, X-ray selected, sample of clusters having $0.17 < z < 0.55$. We derive velocity dispersions and a characteristic radius for each of these clusters, and calculate their virial mass, based upon objective tests to estimate the radial and redshift extent of the cluster members. The Gunn r band luminosity of the clusters is corrected to $z=0$, and the ratio M_v/L to a limiting magnitude of -18.5 (corrected to a total luminosity with a Schechter luminosity function), is found to have a mean value of $283 \pm 27h M_\odot/L_\odot$, comparable to low redshift clusters. The clusters are consistent with having identical M_v/L values (for the corrected luminosities) within the sample variance of $\pm 100 M_\odot/L_\odot$. This is consistent with clusters being approximately isothermal, and being dominated by a relatively red population of galaxies at all the redshifts observed.

The field luminosity density is calculated from the same data set so that we obtain an $(M/L)_c$ in the same redshift range. We find $(M/L)_c = 1160 \pm 130h M_\odot/L_\odot$, where the error is estimated from field-to-field variations. This is consistent, within the errors, with low redshift results, but it should not be compared directly to them because the two are not derived in an identical manner. We conclude that the cluster virial masses universally indicate that $\Omega = 0.25 \pm 0.05$ for clusters at $z \sim 1/3$, or $\Omega_0 = 0.20 \pm 0.04$. Our measurement of Ω is done completely within our data set which diminishes many of the possible selection effects to small corrections to the luminosity, filter bands, and shape of the luminosity function, since they are done identically for the field and cluster galaxies. The concern that relative evolution of field and cluster galaxies could lead to systematic errors is partially constrained here and is addressed elsewhere (Abraham *et al.* 1995 and future papers). Cluster galaxies have largely ceased star formation, so if anything, are likely to be less luminous than field galaxies, meaning that the Ω given here is a mild overestimate.

Complementary to the mean M_v/L value is the average M_v/N_L , which is $3.5 \pm 0.4 \times 10^{12} h^{-1} M_\odot$ to $M_r(0) = -18.5$. and $5.7 \pm 0.7 \times 10^{12}$ to $M_r(0) = -19.5$ (excluding the two binary clusters and MS1621+26). The closure M/N_L values at these two luminosity levels give $\Omega = 0.28 \pm 0.08$. Within the errors the M_v/L and the M_v/N_L (for two different limiting magnitudes) estimates for Ω are consistent with a single value, but, because they are based on differing assumptions about the relative evolution of the cluster and field galaxy population, the sum squared errors in the two methods is used as an indicator of the systematic error resulting from the luminosity normalization, finding that it is 0.09.

Our self-contained technique eliminates many of the systematic errors of the M/L and M/N techniques for Ω estimation, but leaves open the issue whether cluster galaxies are substantially more clustered than the cluster mass. This will be addressed with the same data set in a companion paper.

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Fig. 1.— The redshift vs RA measured from the brightest cluster galaxy direction for the full fields of the 16 clusters. The redshift increases from the vertex of the diagram, and RA is converted to a transverse distance in physical co-ordinates.

Fig. 2.— Redshift vs Declination, as in Figure 1.

Fig. 3.— The redshift histograms in the vicinity of the clusters. The dashed lines indicate the selected cluster redshift range.

Fig. 4.— The sky locations of the galaxies in the redshift range of the cluster. The circle area is proportional to the galaxy’s apparent magnitude. Each box is 1000 arcseconds on a side. The same layout of the clusters as in Figures 1, 2 and 3q is used. North is to the top and East to the left.

Fig. 5.— The corrected luminosities (top row) and normalized colors (bottom row) for cluster galaxies (left column) and field galaxies (right column).

Fig. 6.— The redshift corrected constant overdensity luminosity plotted against the observed σ_1 .

Fig. 7.— The distribution of the deviations from the mean M_v/L (upper left, heavy lines for the 6 well covered clusters), and the M_v/L ratio as a function of total blue fraction, color, mean interior density, velocity dispersions and the redshift.

Fig. 8.— The distribution of the deviations from the mean M_v/N_L (upper left, heavy lines for the 6 well covered clusters) for $M_r(0) = -18.5$. The M_v/N_L ratio is plotted as a function of the quantities the previous figure.

Table 1: Cluster Redshift and Field Limits

Name	z	RA	Dec	z_{lo}	z_{hi}
		h^{-1} Mpc			
A2390	0.2280	6.13	1.11	0.2175	0.2395
MS0016+16	0.5466	2.19	1.87	0.5300	0.5587
MS0302+16	0.4246	1.94	1.73	0.4160	0.4300
MS0440+02	0.1965	3.32	1.02	0.1910	0.2013
MS0451+02	0.2011	4.53	1.57	0.1919	0.2120
MS0451–03	0.5392	2.26	1.87	0.5240	0.5530
MS0839+29	0.1928	3.37	0.96	0.1820	0.2024
MS0906+11	0.1705	1.06	0.86	0.1567	0.1838
MS1006+12	0.2605	1.43	1.20	0.2530	0.2680
MS1008–12	0.3062	1.61	1.36	0.2934	0.3144
MS1224+20	0.3255	1.66	1.35	0.3180	0.3310
MS1231+15	0.2348	1.33	3.16	0.2270	0.2410
MS1358+62	0.3290	1.72	4.27	0.3150	0.3420
MS1455+22	0.2568	1.47	1.21	0.2400	0.2700
MS1512+36	0.3726	5.51	1.70	0.3640	0.3800
MS1621+26	0.4274	2.04	5.15	0.4190	0.4360

Table 2: Velocity Dispersion Validation

Name	σ'_1/σ_1	1 s.d. interval
A2390	1.07	0.99 - 1.11
MS0016+16	1.07	0.93 - 1.15
MS0302+16	0.96	0.77 - 1.09
MS0440+02	0.95	0.60 - 1.14
MS0451+02	0.85	0.67 - 0.98
MS0451–03	0.96	0.84 - 1.01
MS0839+29	0.93	0.55 - 1.10
MS0906+11	0.37	0.22 - 0.49
MS1006+12	0.93	0.74 - 1.00
MS1008–12	1.03	0.90 - 1.09
MS1224+20	0.97	0.85 - 1.04
MS1231+15	1.04	0.95 - 1.10
MS1358+62	0.96	0.91 - 1.02
MS1455+22	0.97	0.85 - 1.09
MS1512+36	1.18	1.00 - 1.32
MS1621+26	0.92	0.83 - 1.00

Table 3: CNOC Cluster Dynamical Parameters

Name	z	N	r_v h^{-1} Mpc	ϵ_r	σ_1 km s^{-1}	ϵ_σ	$\bar{\rho}(r_v)$ $\rho_c(z)$
A2390	0.2280	174	1.972	0.16	1104	63	119
MS0016+16	0.5466	47	0.978	0.12	1234	128	360
MS0302+16	0.4246	28	0.502	0.12	639	92	442
MS0440+02	0.1965	36	1.224	0.16	606	62	99
MS0451+02	0.2011	108	1.371	0.14	1031	73	226
MS0451−03	0.5392	51	0.880	0.10	1371	105	555
MS0839+29	0.1928	42	0.558	0.14	756	111	745
MS0906+11	0.1705	78	0.584	0.04	1888	117	4426
MS1006+12	0.2605	26	0.632	0.06	906	101	738
MS1008−12	0.3062	67	0.575	0.05	1054	107	1113
MS1224+20	0.3255	24	0.478	0.15	802	90	903
MS1231+15	0.2348	73	0.944	0.09	640	65	173
MS1358+62	0.3290	165	1.472	0.13	934	54	128
MS1455+22	0.2568	49	0.621	0.04	1133	151	1203
MS1512+36	0.3726	38	1.019	0.41	690	96	136
MS1621+26	0.4274	98	1.428	0.15	793	55	84

Table 4: CNOC Cluster M_v and L

Name	$\bar{\rho}(r_v)$ $\rho_c(z)$	M_v $h^{-1} \text{M}_\odot$	L $h^{-2} \text{L}_\odot$	M_v/L $h \text{M}_\odot / \text{L}_\odot$	$\epsilon_{M/L}$	M_{200} $h^{-1} \text{M}_\odot$	L_{200} $h^{-2} \text{L}_\odot$
A2390	119	1.7×10^{15}	6.7×10^{12}	248	43	1.3×10^{15}	5.2×10^{12}
MS0016+16	360	1.0×10^{15}	3.6×10^{12}	288	94	1.3×10^{15}	4.8×10^{12}
MS0302+16	442	1.4×10^{14}	6.3×10^{11}	224	108	2.1×10^{14}	9.4×10^{11}
MS0440+02	99	3.1×10^{14}	9.9×10^{11}	312	87	2.2×10^{14}	7.0×10^{11}
MS0451+02	226	1.0×10^{15}	2.8×10^{12}	357	71	1.1×10^{15}	3.0×10^{12}
MS0451−03	555	1.1×10^{15}	2.9×10^{12}	393	102	1.8×10^{15}	4.8×10^{12}
MS0839+29	745	2.2×10^{14}	7.7×10^{11}	285	83	4.2×10^{14}	1.5×10^{12}
MS0906+11	4426	1.4×10^{15}	1.8×10^{12}	800	181	6.6×10^{15}	8.5×10^{12}
MS1006+12	738	3.6×10^{14}	1.2×10^{12}	291	92	6.9×10^{14}	2.3×10^{12}
MS1008−12	1113	4.4×10^{14}	2.0×10^{12}	220	60	1.0×10^{15}	4.7×10^{12}
MS1224+20	903	2.1×10^{14}	1.0×10^{12}	212	114	4.5×10^{14}	2.1×10^{12}
MS1231+15	173	2.7×10^{14}	1.5×10^{12}	176	43	2.5×10^{14}	1.4×10^{12}
MS1358+62	128	8.9×10^{14}	4.5×10^{12}	197	26	7.1×10^{14}	3.6×10^{12}
MS1455+22	1203	5.5×10^{14}	9.3×10^{11}	589	201	1.3×10^{15}	2.3×10^{12}
MS1512+36	136	3.3×10^{14}	1.4×10^{12}	234	156	2.7×10^{14}	1.2×10^{12}
MS1621+26	84	6.2×10^{14}	4.1×10^{12}	151	37	4.0×10^{14}	2.7×10^{12}

Table 5: CNOC Cluster M/N

Name	$M_v/N(-18.5)$	$\epsilon_{M/N}$	$M/N(-19.5)$	$\epsilon_{M/N}$
		$h^{-1} \text{M}_\odot$		
A2390	2.3×10^{12}	3.6×10^{11}	5.7×10^{12}	1.1×10^{12}
MS0016+16	5.8×10^{12}	2.0×10^{12}	5.8×10^{12}	2.0×10^{12}
MS0302+16	2.2×10^{12}	8.6×10^{11}	3.4×10^{12}	2.1×10^{12}
MS0440+02	3.3×10^{12}	9.2×10^{11}	6.8×10^{12}	2.3×10^{12}
MS0451+02	3.8×10^{12}	5.9×10^{11}	7.4×10^{12}	1.7×10^{12}
MS0451-03	7.6×10^{12}	2.0×10^{12}	7.6×10^{12}	2.1×10^{12}
MS0839+29	2.4×10^{12}	7.6×10^{11}	7.7×10^{12}	3.2×10^{12}
MS0906+11	9.6×10^{12}	1.9×10^{12}	2.2×10^{13}	6.1×10^{12}
MS1006+12	5.6×10^{12}	1.8×10^{12}	5.6×10^{12}	1.8×10^{12}
MS1008-12	2.6×10^{12}	6.9×10^{11}	5.0×10^{12}	1.7×10^{12}
MS1224+20	3.0×10^{12}	1.6×10^{12}	4.6×10^{12}	2.1×10^{12}
MS1231+15	1.8×10^{12}	4.5×10^{11}	4.1×10^{12}	1.3×10^{12}
MS1358+62	2.1×10^{12}	2.6×10^{11}	4.2×10^{12}	6.2×10^{11}
MS1455+22	5.7×10^{12}	1.7×10^{12}	1.4×10^{13}	5.8×10^{12}
MS1512+36	2.2×10^{12}	1.5×10^{12}	6.7×10^{12}	4.4×10^{12}
MS1621+26	2.0×10^{12}	4.1×10^{11}	2.6×10^{12}	4.7×10^{11}

Table 6: Average M_v/L and M_v/N_L varying the Absolute Magnitude Limit

$M_r(0)$	M_v/L	$\sigma_{M/L}$	M_v/N_L	$\sigma_{M/N}$
	$h \text{M}_\odot / L_\odot$		$h^{-1} \text{M}_\odot$	
-18.0	310	113	3.3×10^{12}	1.7×10^{12}
-18.5	283	98	3.5×10^{12}	1.6×10^{12}
-19.0	247	80	3.9×10^{12}	1.7×10^{12}
-19.5	241	90	5.7×10^{12}	2.4×10^{12}

Table 7: The Closure M/L and M/N_L Values varying the Absolute Magnitude Limit

$M_r(0)$	$\rho_c/j(> L)$	$\sigma_{M/L}$	$\rho_c/\phi(> L)$	$\sigma_{M/N}$
	$h \text{M}_\odot / L_\odot$		$h^{-1} \text{M}_\odot$	
-18.0	1265	149	1.22×10^{13}	2.1×10^{12}
-18.5	1163	130	1.32×10^{13}	1.9×10^{12}
-19.0	1073	104	1.60×10^{13}	1.5×10^{12}
-19.5	1007	103	2.37×10^{13}	2.6×10^{12}